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On the Irrationality and Transcendence of Rational Powers of *e*

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Abstract

It is argued that a number is irrational if it cannot be represented as the ratio of two integers. The first mathematicians to demonstrate that e is both transcendental and irrational were Lambert and Euler. There have been several extensions of that argument to plausible powers of e and further evidence of the irrationality and transcendence of e since then. This article explores many historical examples when mathematicians have shown that e is irrational and beyond its rational capabilities.

Keywords: Euler's number, irrationality, and transcendence.

Introduction

- **Transcendence of Rational Powers of *e***

• No non-zero polynomial may have a transcendental number as its root, and such numbers can not have rational coefficients. Always remember that just because a number is transcendental does not mean it is unreasonable. There is no such thing as a transcendental number, even if every non-square integer has an irrational square root. Any integer that is not transcendental is called an algebraic number. The first proof that e is transcendental was given by Hermite [22, 23]. After that, Lindemann proved that eα is transcendental when α is a non-zero transcendental number [24, 25], building on these earlier results. Because eiπ = −1, a real number, he also proved that π is transcendental by using this. Weierstrass [26] extended this demonstration, which led to the well-known Lindemann-Weierstrass theorem. This proof was simplified by Gordan [28] and Hilbert [27]. If an algebraic number satisfies a := 0, 1 and b is an irrational but not transcendental algebraic number, then ab is a transcendental number, according to the GelfondSchneider theorem [29], a related theorem. Following this, Baker [30] expanded upon these two theorems much further. A further extension of all these theorems is Schanuel's conjecture [31]. Bernard [32] proved that e is transcendent by using symmetric and multivariate polynomials. The scope of this article is limited to discussing the transcedence of e's reasoning powers. Suppose that the algebraic function ev is defined by the equation c0 + c1ev + c2e2v +... + cnenv = 0 (4.1), where c0 and cn are non-zero integers and ct(0 < t ≤ n) is an integer. When v is a rational number, we may demonstrate that ev is transcendental by using this. An extension of Niven's polynomials is now employed:

fk(x) equals v2k plus 2xk.[(x − 1)...(x − n)]k+1 = 0

Conclusion

This piece examined numerous demonstrations of irrationality and transcendence of rational powers of e established by mathematicians throughout history..

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A rational being's objection that e is

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